The Crystal Structure of Alpha Plutonium Metal*

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Alpha plutonium is monoclinic with 16 atoms in the unit cell which at 21 °C has dimensions

 $a = 6.183, b = 4.822, c = 10.963 \text{ Å}; \beta = 101.79^{\circ}.$

The space group is $P2_1/m$, and all atoms lie in reflection planes.

The atomic position parameters have been determined to a precision of 0.03 Å. Six of the eight kinds of plutonium atom form four short bonds of length $2\cdot57-2\cdot78$ Å and ten long bonds of length $3\cdot19-3\cdot71$ Å. Five short and seven long bonds are formed by the seventh plutonium atom, and three short and thirteen long bonds by the eighth atom.

Introduction

Alpha plutonium is the designation used for the room-temperature form of the element. The phase is stable up to 122 °C, where it transforms to the monoclinic beta modification. No evidence of a phase change at low temperatures has been found.

Several years ago we reported the preliminary findings of a crystal structure investigation of alpha plutonium metal (Zachariasen & Ellinger, 1957). Our subsequent work has led to but minor revisions of the original results, and in this paper we give a final account of our study. Because of the low symmetry of alpha plutonium and because the structure had to be deduced entirely from X-ray powder diffraction data, we have deemed it advisable to describe in some detail the various steps in the investigation. Considerable space is required to give a convincing account of the procedure used to index the diffraction pattern. This step of our study is therefore described in a separate companion article (Zachariasen, 1963).

The experimental data

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All diffraction patterns on which the structure determination was based were taken with Cu $K\alpha$ radiation. The metal was of 99.9% purity with an



Fig. 1. A typical diffractometer trace in the range $30^{\circ} < 2\theta < 40^{\circ}$. The weak line at $2\theta = 38 \cdot 15^{\circ}$ is due to the incident beam hitting the aluminum frame in which the specimen was held. The calculated positions for all possible diffraction lines are shown.

Table 1. Diffraction data $(\sin^2 \theta < 0.320)$

HKL	$10^5 \mathrm{s}$	$\ln^2 heta$	Int	ensity		HKL	105 \$	$\sin^2 \theta$	Int	ensity
	calc.	obs.	obs.	cale.			calc.	obs.	obs.	calc.
001	515		0	1.7		310	0.17125	17125	13	11.4
100	1619		0	0.4		$22\overline{2}$	17249	17257	6	3.9
101	1761		0	0.3		123	17580	17594	7	5.6
002	2061		0	0		204	17706	17701	17	19.9
101	2507		0	0.7		106	17926	17948	39	∫ 2∙3
100	2001		0	0		221	17943	10100	10	38.1
011	2933 3066		0	1.4		215	18175	18169	12	11.1
110	4170		ő	1.0		313	18402		? ?	0.2
111	4312		Õ	3.9		024	18447	18446	70	60.2
102	4427		0	1.0		006	18547	10210	?	6.1
012	4612	4637	7	∫ 5•8		$12\overline{4}$	18573		?	0
003	4637 J	1001	,	1.3		311	18760	18758	3	0.6
102	5058		0	2.4		302	18875	18923	6	∫ 0.3
103	5484		0	0		115	18917 J	10070	0~	2.2
201	6245	6261	7	5-5 6-8		440 999	19078 90995)	19079	95	91.2
200	6477	6482	4	2.2		214	20257	20259	31	20.3
112	6978	6981	27^{-}	33.7		116	20477			1.3
$20\overline{2}$	7045	7043	7	6.8		$20\overline{6}$	20544	20547	8	2.5
013	7188	7187	56	56.3		$31\overline{4}$	20888		<4	0.8
103	7376	7377	13	16.6		016	21098	21086	17	10.1
113	7687	7690	122	128.7		312	21426		0?	0.8
004	8943	27737	04 105	64.9		124	21559	21558	16	10.8
104	8369	0244	105	98.4		305	21804	91099	0?	0
$21\overline{\overline{1}}$	8796	8804	12	13.9		106	21930	21932	10	0.2
$20\overline{3}$	8874	8878	152	142.1		303	22400 22571	22563	8	0°3 5•0
210	9028	9032	15	11.3		$12\bar{5}$	22836	22842	11	6.6
$21\overline{2}$	9596	9601	11	11.6		030	22960		0	Õ
113	9927	9922	12	10.7		025	23084			(0.6
202	10031	10011	< 6	1.1		205	23090	23091	12	$\left\{ 0.5\right.$
040 911	10204	10211	246	270.6		216	23095 J		0	6.8
021	10230	10290	204	301.4		031	23475		0	0.2
014	10794	10794	121	105.6		$32\overline{1}$	23538		0	(2.1
$11\overline{4}$	10920	10924	86	72.6			24251	24233	4	1 1.8
104	11355	11352	15	14.6		315	24405	24405	10	6.1
213	11425	11425	42	$39 \cdot 2$		130	24579 \	94601	96	ſŌ
204	11734	11737	8	7.7		322	24599 ∫	24001	30	(ે 30∙6
120	11020		0	0		131	24721		?	0
022	12265		0	Ŏ		320 401	24778		?	2.6
212	12582	12582)		(18.4		116	24951	94940	0	0.4
$10\overline{5}$	12632	12628	25	8.4		$40\overline{2}$	24984	21010	0	$1 - \frac{4}{0.2}$
121	12711	,	0	0.1		032	25021		0?	0.5
005	12880		0	0.8		313	25122	95990	e	(0.7
122	13137		0	0.4		007	25245 ∫	40228	0	〔 2·9
203	13354	19000)	0	1.5		131	25467		0	0.3
301	13969	13900	10	13.2	Í	215	25641	25630	21	25.2
$21\frac{1}{4}$	14285	13901)	02	(3.7		225 139	20828	25829	11	6.6
$30\overline{2}$	14395	14399	37	36.0		400	25909		: ?	0.3
3 0 <u>0</u>	14574	14583	5	3.2		$32\overline{3}$	26055	0.00 - 0	•	(5.9
122	14631		0	0.4		$40\overline{3}$	26066	26073	9	1 0.8
02 <u>3</u>	14841		0	0.6		306	26402			5.3
115	15183		0	$2 \cdot 1$		321	26413	26488	8	3.3
123	15340		0	0	Ì	$20\overline{7}$	26496			2.2
015 90F	15431	15431	5	2.5		125	26570	26573	14	9.8
200	15024	12033	13	7.5		117	26802	26797	21	20.7
213	15905	15869	16	1 0·0 1 1.1		304	27297		0?	0.6
301	16209	16208	13	3.6		102	21301 27489 1		0?	4·3
105	16366	16362	16	10.8		412	27535	27526	52	35.0
$22\overline{1}$	16449		?	3.4		033	27597			7.4
311	16520	16518	13	$5 \cdot 4$	ļ	017	27796	27783	45	41.1
220	16681	10051	0	1.2	[224	27910)	27900	30	∫ 18.9
312	10946	16954	19	18.0	I	401	27917 ∫	21000	30	12

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				Table	1 (cont.)				
HKL	$10^5 { m \ s}$	$ in^2 \theta $	\mathbf{Int}	ensity	HKL	10 ⁵ s	$n^2 \theta$	Int	ensity
	calc.	obs.	obs.	calc.		calc.	obs.	obs.	calc.
$13\overline{3}$ $12\overline{6}$ $40\overline{4}$	$28096 \\ 28130 \\ 28179$	28090	23	$\left\{\begin{array}{c}17\cdot8\\2\cdot1\\0\cdot7\end{array}\right.$	$\begin{array}{c} 314\\ 23\overline{2}\\ 133\end{array}$	29848 30005 30336	29839	12 ?	$15.2 \\ 2.0 \\ 1.9$
$\frac{404}{32\overline{4}}$	28179 28460 28541		0? 0	3·0 0·4	411 231	30468 30699	$30479 \\ 30705$	$\ddot{7}$ 61	1·3 53·5
$\begin{array}{c} 41\overline{3} \\ 026 \end{array}$	$28617 \\ 28751$	$\begin{array}{c} 28624\\ 28759\\ \end{array}$	6 10	0·5 5·8	$\begin{array}{c} 41\overline{4}\\ 22\overline{6}\\ 402\end{array}$	30730 30748		? ?	3·1 2·7
$\frac{316}{217}$	28953 29047 29079	28957	55 ? ?	$45.8 \\ 2.1 \\ 0.3$	$ \begin{array}{r} 402 \\ 034 \\ 405 \end{array} $	30956 31203 31323)	31213	24	1.6 19.4
$\begin{array}{c} 322\\ 23\overline{1}\\ 230\end{array}$	29205 29437		? 0	$2 \cdot 1$ 1 \cdot 9	$13\overline{4}$ $10\overline{8}$	31329 } 31605	31319	17 ?	$\begin{cases} 13.5\\ 2.4 \end{cases}$
$\begin{array}{c} 107 \\ 206 \end{array}$	$29477 \\ 29504$		0 0	0·7 0·7	233	31834	31838	12	7.9

Table 2. Diffraction data ($0.320 < \sin^2 \theta < 0.600$)

	10 ⁴ si	$\mathbf{n}^2 \ heta$	Inter	isity		10 ⁴ si	$\ln^2 heta$	Inte	ensity
HKL	calc.	obs.	obs.	calc.	HKL	calc.	obs.	obs.	calc.
323	3278	3277	4	5.6	235	4605	4606	10	10.5
232	3299)	0200	_	(4.1	309	4623	1000	10	6.3
305	3305	3299	7	1 3.5	405	4626 🕻	4023	16	10 ⋅8
$20\overline{8}$	3348	0070		3.0	137	4721	4716	11	<u>`</u> 8∙9
412	3351	3352	21	1 20.9	128	4778			(2 ⋅8
$11\overline{8}$	3416	3414	4	2.5	431	4789 }	4792	25	{ 9∙4
134	3432)			(3.2	$43\overline{2}$	4794 J			15.6
$12\overline{7}$	3446	3445	8	$\left\{ 2 \cdot 0 \right\}$	037	4821	4818	24	18.5
$31\overline{7}$	3453			4.4	241	4856	4851	11	$8 \cdot 3$
403	3 50 3 ´	3501	4	4.6	319	4878)			6.4
027	3545)	0551		3.3	415	4881 }	4879	25	$\left\{ 2 \cdot 8 \right\}$
018	3552	3991	11	〔 5·0	512	4883			10.8
$32\overline{6}$	3661 1	9664	11		044	4906	4906	21	13.4
$22\overline{7}$	3670 👔	3004	11	12.8	$52\overline{1}$	4934)			10·3
$33\overline{2}$	3736 [´]	3736	5	` 5∙1	336	4936 }	4936	40	$\{ 21.6 \}$
330	3753)			(3.7	1,0,10	4941)			8.5
413	3758	3755	8	{ 1.4	243	4969	4966	26	$21 \cdot 1$
108	3758			1.9	334	5026	5026	10	$7 \cdot 4$
$23\overline{5}$	3 858 ´	3859	4	3.3	1,1,10	5196 \	5916	1	∫ 3·1
$50\overline{1}$	3913	3913	8	$7 \cdot 2$	144	5217 ∫	5210	Ŧ) 3∙0
118	4013			$4 \cdot 2$	521	5307	5206	8	∫ 5∙5
234	4067)	4009	11	1 9.8	2,1,10	5308 ∫	0000	0	<u>)</u> 5∙3
040	4082 Ì	4000	44	∖ 31·6	432	5392	5391	13	10.8
036	4151	4152	5	3.5	525	5423	5424	34	37.0
$51\overline{1}$	4168	4179	0	∫ 7·8	$34\overline{2}$	5521	5521	13	$9 \cdot 8$
$12\vec{8}$	4181 🐧	4174	9	<u>ો</u> 3∙1	228	5563	5558	32	$32 \cdot 1$
501	4287			3.5	$32\overline{9}$	5643			(10.3
$41\overline{7}$	4325	4997	. 9	∫ 2.5	245	5644	5642	34	$\left\{ 2\cdot 3\right\}$
325	4326)	4541	0	∖ 4·6	425	5646)			17.6
$23\overline{6}$	4350	4351	4	2.5	428	5714 \	5719	7	∫ 3∙9
$22\overline{8}$	4368	4365	5	4 ·0	145	5718 ∫	5712	•	<u></u>
$50\overline{5}$	4403	4404	94	(23·3	3,1,10	5745	5740	8	$5 \cdot 3$
$21\overline{9}$	4404	4404	94	\ 10 ∙5	600	5830	5829	5	$4 \cdot 0$
423	4523 j			(6·4	244	5852	5848	7	$7 \cdot 3$
511	4542	4541	28	{ 3.5	611	5912			$3 \cdot 2$
208	4542			 19 ∙9	1,2,10	5961	5958	13	14.5

isotopic composition corresponding to an atomic weight of 239.05.

Numerous photographic patterns were made from samples of metal filings, but recordings prepared with a Philips diffractometer proved more useful. Metal buttons with electropolished surface of $1\frac{1}{2}$ cm diameter served as samples for the diffractometer data. The patterns with the greatest resolution were obtained when the polished surface layers of the metal were removed by repeated etching. The quality of the data is illustrated in Fig. 1, which shows an unretouched reproduction of the diffraction pattern for the range $30^{\circ} \leq 2\theta \leq 40^{\circ}$. It is seen that diffraction lines of comparable intensity are resolved when the glancing angles differ by as little as 0.05° , and that the $\alpha_1 \alpha_2$ doublet begins to resolve into two peaks at $\theta = 18^{\circ}$.

 α_a αь an Δ α1 α_2

 α_3 φ

For the angular range shown in Fig. 1 the scattering angles could be measured to better than 0.01° and hence the maximum error in the experimental values of $\sin^2 \theta$ is 5×10^{-5} . In order to get enough intensity at larger scattering angles it was necessary to use wider slits with a corresponding loss of resolution. Altogether about 165 α_1 -lines could be measured in the pattern.

The coefficients of thermal expansion were determined from photographic data in the back reflection region. In these experiments the plutonium filings were mixed with powdered silver of high purity. The silver diffraction lines were used to find the correction curve for the measured glancing angles due to finite sample size and absorption.

Only insignificant variations in intensity caused by preferred orientation were noted. Plutonium oxidizes readily, and in spite of precautions it was difficult to avoid oxide formation on the surface. Accordingly, weak and diffuse oxide lines were frequently observed.

The unit cell

All attempts to index the diffraction pattern on the basis of orthorhombic and higher symmetry failed. The companion article describes in detail how the diffraction pattern was indexed in accordance with monoclinic symmetry, and the results of the indexing are shown in the accompanying tables.

In the diffraction pattern of alpha plutonium taken with Cu $K\alpha$ radiation are more than 900 possible

Table	3.	Diffre	iction	data
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D 1	en	•
Back	reflection	region

	104 si	$n^2 \theta$	Inte	\Rightarrow nsity
HKL	calc.	obs.	obs.	calc.
$71\overline{4}$ $71\overline{1}$ $54\overline{1}$	7969 7980 } 7995	7972 7994	110	$\left\{ egin{array}{c} 54 \\ 15 \\ 25 \end{array} ight.$
$\begin{array}{r} 254 \\ 53\overline{8} \\ \hline \end{array}$	$\left. \begin{array}{c} 8148\\ 8148 \end{array} \right\}$	8150	45	$\left\{\begin{array}{c}18\\16\end{array}\right.$
710	8190	8192	35	21
419	8303	8364	65	60
040 040	8480	8485	110	108
248	8024	8620	95	102
$255 \\ 34\overline{9} \\ 445 \\ 3,0,\overline{13}$	8687 8704 8707 8708	8685 8705	210	$\left\{\begin{array}{c} 27\\ 34\\ 58\\ 61\end{array}\right.$
$\begin{array}{c} 45\overline{1} \\ 45\overline{2} \\ 057 \end{array}$	8871) 8876 } 8902	8879 8903	130	$\begin{cases} 28 \\ 47 \\ 57 \end{cases}$
$35\overline{6}$ 1,4, $\overline{10}$	$\left. \begin{array}{c} 9018\\ 9022 \end{array} \right\}$	9017	130	$\left\{\begin{array}{c}72\\56\end{array}\right.$
1,3,11 354	9102 9108	9102	80	$\left\{\begin{array}{c}26\\31\end{array}\right.$
708	9141	9141	70	4 5
703 060	$\left. \begin{array}{c} 9182\\ 9184 \end{array} \right\}$	9184	180	$\left\{\begin{array}{c} 38\\89\end{array}\right.$
616	9283	9283	80	64
$\begin{array}{r} 452\\ 5,1,\overline{12} \end{array}$	$\left. \begin{array}{c} 9473 \\ 9483 \end{array} \right\}$	9479	150	$\begin{cases} 55 \\ 98 \end{cases}$
3,2,13	9729	9729	300	263

diffraction lines. A great many of these are too weak to be observed, and many lines are superimposed.

All possible reflections in the range $\sin^2\theta \le 0.320$ are listed in Table 1 together with those actually observed. Table 2 covers the range $0.320 \le \sin^2 \theta \le 0.600$; but all diffraction lines with small calculated intensity have been omitted. Data for the strongest diffraction lines (as calculated) in the back reflection region are given in Table 3. The results of measurements of selected strong lines in the back reflection area at different temperatures are shown in Table 4.

The unit cell dimensions as deduced from the data of Table 4 are listed in Table 5. The measured density

Table 4. Thermal expansion data $10^4 \sin^2 \theta$

	21	°C	51	°C	96	°C	104	°C
	\sim		\sim				$ \longrightarrow $	_
HKL	calc.	obs.	calc.	obs.	calc.	obs.	calc.	obs.
419	8363	8364	8341	8343	8307	8308	8297	8297
545	8485	8485	8458	8459	8408	8406	8397	8395
248	8623	8620	8599	8599	8556	8550*	8544	8543*
$70\overline{8}$	9141	9141	9115	9116	9072)		90671	
703	9182)	0104	9152)	0150	9101	9099	9090	9077
060	9184 <i>(</i>	9184	9153	915Z	9090		9071	
616	9283 [´]	9283	$9254^{'}$	9254	9208	9207	9196	9193
452	9473)	0470	9441)	0461	9381	9376	9364	9365
$5,1,\overline{12}$	9483∫	9419	9461 j	9401	9428	9427	9424	9422
$3,2,\overline{13}$	9729	9729	9709	9708	9677	9678	9672	9672
640	9911		9878		9817	9817	9802	9802

* Coincidence with silver line.

Table 5. Cell dimensions (Å) and calculated densities (g.cm⁻³)

	21 °C	51 °C	96 °C	104 °C
a	6.183 ± 0.001	6.194	6.211	6.214
b	$4 \cdot 822 \pm 0 \cdot 001$	4.831	4.847	4.852
с	$10{\cdot}963 \pm 0{\cdot}001$	10.973	10.987	10.989
β	$101.79 \pm 0.01^\circ$	101·78°	101·77°	101·75°
e	19.86 ± 0.01	19.77	19.62	19.59

of 19.77 g.cm⁻³ corresponds to 16 plutonium atoms per unit cell. The calculated densities with this cell content are also shown in Table 5.

The experimental results for the linear coefficients of thermal expansion in the temperature range 21-104 °C are presented in Table 6. The symbols $\alpha_a, \alpha_b, \alpha_c$ are the linear coefficients in the directions of the crystallographic axes while Δ denotes the

Table	э б.	Expansion	coefficients
	•••	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	000,1000,000

	$(\times 10^{6})$		
21°–51°	21°-96°	21°–104°	Mean 21°-100°
59	60	60	60 + 1
57	69	75	72 + 3
30	29	29	29 + 1
-6	-5	-8	-7 ± 2
			62 ± 1
			72 ± 3
			29 ± 1
			13 + 2



Fig. 2. The generalized projection $\int_{0}^{1} \varrho(1 + \sin 2\pi y) dy$ as evaluated on XRAC.

change in angle β (measured in radians) per degree rise in temperature. These quantities are related to the principal coefficients α_1 , α_2 , α_3 and the angle φ describing the orientation of the tensor ellipsoid by the equations (see Zachariasen & Ellinger, 1959)

$$\begin{aligned} \alpha_1 - \alpha_3 &= -\Delta/\sin\beta\,\cos\left(2\varphi - \beta\right) \\ \tan\left(2\varphi - \beta\right) &= (\alpha_a - \alpha_c)/\Delta \\ \alpha_1 + \alpha_3 &= \alpha_a + \alpha_c + \Delta\,\cot{a}\beta \ (1) \end{aligned}$$

where φ is the angle between the x-axis of the principal tensor ellipsoid and the crystallographic *a*-axis measured in the obtuse angle β . It is seen that the direction of minimum expansion (α_3) is nearly parallel to the *c*-axis.

Determination of the structure

No systematic absences occur for reflections (H0L)and (HKL). Reflections (010), (030), (050) are not present, suggesting a screw axis along Y; but the evidence is not conclusive in view of the many accidental absences.

The data of Tables 1–3 show that the condition $|F_{HKL}|/f = |F_{H, K+2, L}|/f$ is fulfilled for any set H, K, L, and that the reflections (020), (040), (060) are exceptionally strong. These observations require the space group to be $P2_1/m$ with all atoms lying in the reflection planes: $\pm (x, \frac{1}{4}, z)$.

Approximate values for the sixteen position parameters of the structure were deduced by means of the probable validity of the relation (see Zachariasen, 1952a)

$$S_{H+H'} = S_H S_{H'} \tag{2}$$

for strong reflections (where the symbol S denotes the structure factor sign).

Because of the relation

$$F_{HKL}/f = -F_{H, K+2, L}/f \tag{3}$$

it was sufficient to consider reflections (H0L) and (H1L). Without loss of generality the structure factor signs for the three strong reflections (203), (211), (505) were assumed to be positive. By application of equation (2) it was possible to determine uniquely the amplitude signs for 41 strong reflections while the signs for an additional set of 10 strong reflections could be expressed in terms of the symbol b. The results of these considerations are given in Table 7.

Table 7. Amplitude signs

H0L	sign	H1L	sign	H1L	sign
004		013	+	314	b
0,0,10	-b	014	+	$41\bar{2}$	+
0,0,11	+	017	—	411	-b
1,0,10	+	0,1,11	+	412	
104	-	$11\overline{7}$	+	419	+
105	b	114	b^*	5,1,12	+
$20\bar{5}$	b	113	-	519	
$20\overline{3}$	+*	1,1,11	+	518	
201	-	2,1,10	+	$51\overline{1}$	+
204	-	219	-b	512	-
208	+	$21\overline{3}$	—	616	+
3,0,13	+	211	+*	$71\overline{4}$	+
$30\overline{2}$	+	214	_	710	-
403	b	215	\rightarrow		
$50\bar{9}$	_	3,1,10	+		
$50\overline{5}$	+*	$31\overline{9}$	+		
$50\overline{1}$	-	317	Ь		
600	b	316	-		
$70\bar{8}$	+	$31\overline{2}$	+		

* Sign assumed.

Next, generalized Fourier syntheses

$$\int_{0}^{1} \varrho \left(1 + \sin 2\pi y\right) dy \tag{4}$$

were evaluated using the signs from Table 7, setting first b=+1 and then b=-1. The Fourier terms involving the symbol b, being fewer in number and smaller in magnitude, had only a modulating effect on the projection. However, it was immediately apparent that b=+1 was the correct choice, for it gave an electron distribution with the required number of eight maxima of nearly equal density.

The approximate atomic positions having been found, additional Fourier terms could be included and successive approximations made. Fig. 2 shows the XRAC photograph of the final generalized electron projection. The coordinates deduced from this Fourier synthesis are those reported in our preliminary communication.

Since the generalized Fourier syntheses utilized only a small number of terms (41 H0L terms and 53 H1L terms), it was to be expected that the leastsquare method of refinement would give better position parameters.

The least-square refinements were made by using the Busing-Levi IBM-704 program. The Fourier results served as starting point, and an isotropic temperature factor B=0.2 Å² was assumed and not varied in the course of the refinement. For the *f*-curve of plutonium the effective scattering power, *f*, was taken to be

$$f = [(f_0 - \Delta')^2 + \Delta''^2]^{\frac{1}{2}}$$
(5)

where f_0 is the Fermi-Thomas value (Thomas, Umeda & King, 1958) and Δ' and Δ'' , the empirical dispersion corrections reported by Roof (1961). The refinements (based on 97 reflections for which the structure factors could be reliably measured) gave R=0.07 and the parameter values listed in Table 8. The reliability factor and standard errors are as low as the quality of the data justifies.

Table 8. Position parameters from least-square refinement

x	z
0.345 ± 0.004	0.162 ± 0.002
0.767 ± 0.004	0.168 ± 0.002
0.128 ± 0.004	0.340 ± 0.003
0.657 ± 0.005	0.457 ± 0.003
0.025 ± 0.005	0.618 ± 0.003
0.473 ± 0.004	0.653 ± 0.002
0.328 ± 0.004	0.926 ± 0.002
0.869 ± 0.004	0.894 ± 0.002
	$x \\ 0.345 \pm 0.004 \\ 0.767 \pm 0.004 \\ 0.128 \pm 0.004 \\ 0.657 \pm 0.005 \\ 0.025 \pm 0.005 \\ 0.473 \pm 0.004 \\ 0.328 \pm 0.004 \\ 0.869 \pm 0.004 \\ 0.869 \pm 0.004 \\ 0.801 \\ 0.004 \\ $

The last columns of Tables 1-3 give the intensities as calculated from the structure by the relation

$$I \propto p|F|^2 (1 + \cos^2 2\theta) / \sin^2 \theta \cos \theta .$$
 (6)

It should be emphasized that Tables 2 and 3 include all observed α_1 -lines in the pertinent angular ranges

and all reflections with appreciable calculated intensity.

Discussion of the structure

The structure as viewed along the *b*-axis is shown in Fig. 3. The atoms are arranged in pseudo-hexagonal layers normal to the *b*-axis and $2\cdot411$ Å apart. Each atom forms six bonds with atoms in the same reflection plane and six to ten bonds with atoms of adjacent layers. The distortion from hexagonal configuration in the reflection plane is so great that the bond lengths in this plane range from $2\cdot57$ to $3\cdot71$ Å.



Fig. 3. Projection of the structure on the (010) plane. Filled circles represent atoms at $y = \frac{1}{4}$, open circles atoms at $y = -\frac{1}{4}$. Only the short bonds are indicated in the projection.

Table 9 lists all individual bond distances. Because other interatomic distances exceed 4 Å, the value of 3.71 Å is quite well defined as the extreme radius for the first coordination sphere. It is seen from Table 9 that the various bonds fall naturally into

Table 9. Individual bond lengths (Å)

I		I	ſ	11	Ι	I	V
VII	2.57	Ī	2.60	ī	2.58	v	2.58
III	2.58	III	2.61	ĪI	2.61	VI	2.64
II	2.60	VII	2.64*	v	2.66*	VI	2.74*
\mathbf{VIII}	2.76*	\mathbf{VIII}	3.19	v	3.24	III	3.26
VI	3.21*	IV	3.37	IV	3.26	III	3.26*
\mathbf{IV}	3.41	\mathbf{VII}	3.39	IV	3.26*	v	3.32*
\mathbf{VII}	3.42*	v	3.42*	IV	3.42	IV	3.35*
II	3.59	VIII	3.46*	VI	3.44*	II	3.37
\mathbf{VIII}	3.71	I	3.59	\mathbf{VIII}	3.52*	I	3.41
		VI	3.62*	\mathbf{VI}	3.65	III	3.42
V	7	v	I		II	VI	II
$\frac{V}{IV}$	2.58	\overbrace{IV}^{V}	I 2·64		11 2·57		$\frac{11}{2.76*}$
	2·58 2·66*	$\overbrace{V}{V}_{V}$	I 2.64 2.72		11 2·57 2·64*	$\overbrace{\substack{I\\VII}}^{VI}$	$ 11 \overline{2.76*} 2.78 $
V IV III VI	2·58 2·66* 2·72	\overbrace{V}_{V}	I 2.64 2.72 2.74*		11 2·57 2·64* 2·78	$\overbrace{I}^{VI}_{VII}_{II}$	$ \underbrace{\begin{array}{c}11\\2.76*\\2.78\\3.19\end{array}} $
IV IV III VI III	2.58 2.66* 2.72 3.24	$\overbrace{\substack{IV\\V\\IV\\I}}^{V}$	I 2.64 2.72 2.74* 3.21*	I I VIII VI	11 2·57 2·64* 2·78 3·30	VI I VII VII VI	11 2·76* 2·78 3·19 3·21
V IV III VI III IV	2·58 2·66* 2·72 3·24 3·32*	\overbrace{V}^{V} \overbrace{V}^{V} IV I V I V I	$ \begin{array}{c} 1 \\ 2 \cdot 64 \\ 2 \cdot 72 \\ 2 \cdot 74* \\ 3 \cdot 21* \\ 3 \cdot 21 \end{array} $	I I VIII VI II	11 2·57 2·64* 2·78 3·30 3·39	VI I VII VI VI VI V	11 2.76* 2.78 3.19 3.21 3.36
V IV III VI III VV VIII	2·58 2·66* 2·72 3·24 3·32* 3·36	\overbrace{V}^{V} \overbrace{V}^{V} IV V V V V V V V V V	I 2·64 2·72 2·74* 3·21* 3·21 3·30	I I VIII VI II VII	11 2.57 2.64* 2.78 3.30 3.39 3.40*	VI I VII VI VI VI VI	11 2.76* 2.78 3.19 3.21 3.36 3.44
IV IV III VI III IV VIII II	2·58 2·66* 2·72 3·24 3·32* 3·36 3·42*	\overbrace{V}^{V} \overbrace{V}^{V} IV V V V V I V I V I I I	I 2.64 2.72 2.74* 3.21* 3.21 3.30 3.44*	I VIII VIII VIII I I	11 2·57 2·64* 2·78 3·30 3·39 3·40* 3·42*	VI I VII VI VI VI VII II	11 2.76* 2.78 3.19 3.21 3.36 3.44 3.46*
V IV III VI III IV VIII II V	2.58 2.66* 2.72 3.24 3.32* 3.36 3.42* 3.50*	V V V V V V V V V V	I 2.64 2.72 2.74* 3.21* 3.21 3.30 3.44* 3.51	I I VIII VI I VIII VIII VIII	II 2.57 2.64* 2.78 3.30 3.39 3.40* 3.42* 3.44	VI I VII VI VI VII VII VII	11 2·76* 2·78 3·19 3·21 3·36 3·44 3·46* 3·48*
V IV III VI III V VIII V VI	2.58 2.66* 2.72 3.24 3.32* 3.36 3.42* 3.50* 3.51	$\begin{array}{c} V\\ \overbrace{IV}\\ V\\ IV\\ I\\ VIII\\ VIII\\ III\\ V\\ II \end{array}$	I 2.64 2.72 2.74* 3.21* 3.30 3.44* 3.51 3.62*	VI I VIII VI I VIII VIII VIII	II 2.57 2.64* 2.78 3.30 3.39 3.40* 3.42* 3.42* 3.44 3.48*	VI I VII VII VI VII VII VII VIII	II 2.76* 2.78 3.19 3.21 3.36 3.44 3.46* 3.48* 3.50*
V IV III VI III VV VIII VIII VI VI VI	2.58 2.66* 2.72 3.24 3.32* 3.36 3.42* 3.50* 3.51 3.51	\overbrace{V}^{V} \overbrace{V}^{V} IV IV $VIII$ $VIII$ VII III III III	$I \\ 2.64 \\ 2.72 \\ 2.74 \\ 3.21 \\ 3.21 \\ 3.30 \\ 3.44 \\ 3.51 \\ 3.62 \\ 3.65$	VI I VIII VI VI VI VIII VIII VIII V	II 2.57 2.64* 2.78 3.30 3.40* 3.42* 3.42* 3.44 3.48* 3.51	VI I VII VI VI VII VII VII VIII III	$\begin{array}{c} 11\\ \hline 2.76*\\ 2.78\\ 3.19\\ 3.21\\ 3.36\\ 3.44\\ 3.46*\\ 3.48*\\ 3.50*\\ 3.52* \end{array}$

* Denotes two bonds (with atoms lying in adjacent reflection planes).

two groups: 'short' bonds of length 2.57-2.78 Å and 'long' bonds of length 3.19-3.71 Å. The distribution of the bonds among the two groups for the eight distinct kinds of plutonium atoms is shown in Table 10.

Table 10. 'Short' and 'long' bonds (Å)

					All	bonds
Atom	Short bonds		Long bonds			Mean
\mathbf{type}	No.	\mathbf{Range}	No.	Range	No.	length
Ι	5	2.57 - 2.76	7	$3 \cdot 21 - 3 \cdot 71$	12	3.10
II	4	$2 \cdot 60 - 2 \cdot 64$	10	3.19 - 3.62	14	3.21
III	4	2.58 - 2.66	10	$3 \cdot 24 - 3 \cdot 65$	14	3.18
IV	4	2.58 - 2.74	10	$3 \cdot 26 - 3 \cdot 42$	14	3.13
v	4	2.58 - 2.72	10	$3 \cdot 24 - 3 \cdot 51$	14	3.19
VI	4	2.64 - 2.74	10	$3 \cdot 21 - 3 \cdot 65$	14	3.22
\mathbf{VII}	4	2.57 - 2.78	10	3.30 - 3.51	14	3.12
\mathbf{VIII}	3	2.76 - 2.78	13	3.19 - 3.71	16	3.32

Similar well defined short chemical bonds, presumably of covalent nature, have previously been observed for other heavy metals. In the α -uranium structure (Jacob & Warren, 1937) there are four short bonds of length 2.76-2.82 Å. Four short bonds, 2.60-2.64 Å long, were also found in the structure of α -neptunium (Zachariasen, 1952b), and there are four short bonds of length 2.72 Å in β -neptunium (Zachariasen, 1952c).

The five short bonds formed by Pu(I) are directed approximately towards the corners of a trigonal bipyramid, the three short bonds of Pu(VIII) approximately towards the corners of an equilateral triangle. As seen from Fig. 3, the four short bonds formed by atoms II-VII tend to lie within one hemisphere, and this is true also for the four short bonds in the uranium and neptunium structures.

The total numbers of bonds (short and long) formed are 12 for Pu(I), 16 for Pu(VIII) and 14 for the six other types of plutonium atoms, and the mean bond lengths are respectively $3\cdot10$, $3\cdot32$ and $3\cdot18$ Å. When corrections are made for the effect of coordination number, these values give $1\cdot58$ Å for the metallic radius of plutonium in the α -phase. As has been discussed in detail in another publication (Zachariasen, 1961), the plutonium radius is appreciably smaller in the α , β , γ , ε phases than in the δ and δ' phases.

The main features of the observed anisotropy in

the thermal expansion can be explained by means of the strong, short bonds. Let it be assumed that all short bonds are of equal strength, let the directions of the bonds be described by unit vectors u_i , and let us form the dyadic $\varphi = \sum u_i u_i$ where the sum is extended over all short bonds. The direction s of a principal axis of the tensor is given by the condition that $s. \varphi. s$ be an extremum. The direction of maximum thermal expansion tends to be parallel and the direction of minimum thermal expansion normal to the chemical bonds. Accordingly one should expect the tensor φ and the thermal expansion tensor to have parallel orientations. Indeed, this is true. The quantity $s. \varphi. s$ has its maximum value when s is along the b-axis, its minimum value when s is approximately parallel to the *c*-axis, and the two long principal axes

are of nearly equal length. It has not been possible to find a simple structural relationship between alpha and beta plutonium, thus indicating that a major rearrangement of the atoms is involved in the $\alpha \rightleftharpoons \beta$ transition.

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